Drawing hypergraphs using NURBS curves

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Content

Introduction

Things done before

As a student of computer science I thought about how to draw graphs.

⇒ student research project: **Gravel**, an editor for graphs

- focus on scientific illustrations and convenient editing
- export especially for TEX (vector graphics)

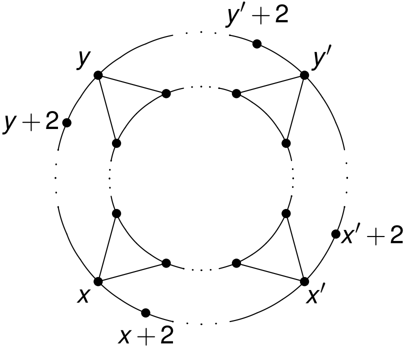


Figure: graph generated with Gravel, from Schiermeyer et al.

Introduction

What am I going to talk about?

working on Gravel I noticed

- some other editors for graphs available
- none supported editing of hypergraphs

$\Rightarrow$ drawing hyperedges with other tools

$\Rightarrow$ no convenient editing

$\Rightarrow$ **diploma thesis** about drawing and editing hypergraphs

Drawing a hypergraph

How do scientists usually draw hypergraphs?

common drawing: **the subset standard**

- nodes are drawn as dots (as in graphs)
- usually nodes are placed first
- hyperedge is a subset of nodes

$\Rightarrow$ drawn as a “cloud” surrounding all its nodes → blackboard

$\Rightarrow$ handle these “smooth” curves (on a computer)

Requirements of curves

...for drawing and editing of a hyperedge

a curve representing a hyperedge should be

- numerically stable
- easy to create
- convenient and interactive editible, e.g. by affin-linear trasnformations
- edited just locally (if most of the curve is already done)
- **periodic**, so that it
  - looks “smooth”, without rough edges
  - outlines all nodes

common in computer graphics: B-splines & NURBS
but they are usually not periodic.

B-splines – definition

a very short introduction of B-splines

a B-spline curve $B(u), u \in [a,b]$ consists of

- a polynomial degree d
  $\Rightarrow$ smoothness
- $n+1$ control points $P_0,\ldots,P_n$
  $\Rightarrow$ form
- a knot vector $(t_i)_{i=0}^m$ with $m = n+d+1$ (nondecreasing)
  $\Rightarrow$ recursive B-spline basis functions $N_{i,d}(u), i = 0,\ldots,n$
  - piecewise nonnegative polynomial functions
  - model distribution of the P_i “along the curve”

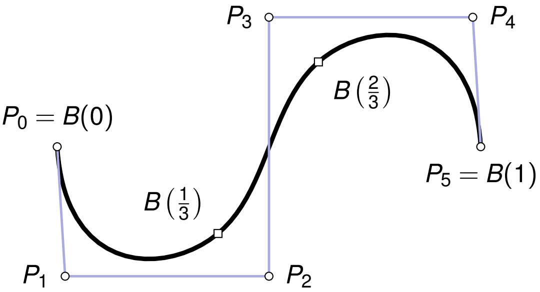
with those we get the **B-spline curve**

$$B(u) = \sum_{i=0}^{n} N_{i,d}(u)P_i, \quad u \in [t_d, t_{m-d}] = [a,b]$$

B-splines – Example

a simple B-spline curve

- degree 3
- $n = 5 \Rightarrow$ six control points
- knot vector $(t_i)_{i=0}^{9} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 1 & 1 & 1 \end{pmatrix}$



P_1 has local influence: $[t_1, t_5) = [0, \frac{2}{3})$

B-splines – properties

nice features we obtain by using B-spline curves

- P_0 and P_n are interpolated, if $t_0 = t_d = a$ and $t_{m-d} = t_m = b$
- P_i only has **local** influence to the curve
- Let p be the maximum multiplicity of a knot from t_{d+1} to t_{m-d-1}

$\Rightarrow$ $B(u)$ is "smooth", because it is $d-p$-times differentiable

try to reach $p = 1$!

with $B(u)$ get a curve that

- is numerical stable
- has easy computable derivatives
- is a piecewise polynomial

But we can't form circles!

NURBS

Non **U**niform **R**ational **B**-**S**plines

additional weight $w_i > 0$ for each control point P_i

$\Rightarrow$ different influence of each P_i to the curve

$\Rightarrow$ new NURBS basis functions $R_{i,d}(u)$, piecewise rational polynoms (based on knot vector and weights)

with that modification

- NURBS curve $C(u) = \sum_{i=0}^{n} R_{i,d}(u) P_i$ is a piecewise rational polynomial
- circles are possbile

most important

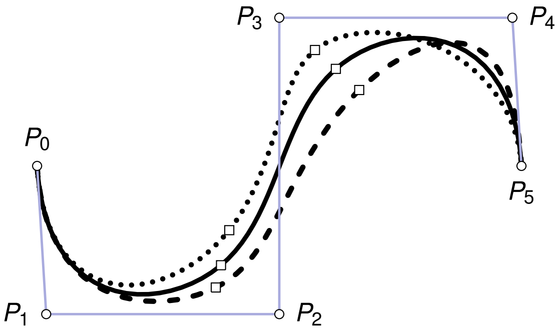
- B-spline properties also apply for NURBS
- using B-spline algorithms by calculation in **homogeneous coordinates**

NURBS – example

showing different influences of a control point to the curve

variation of weight w_3 from the last example with

- $w_3 = 2$ (dotted line)
- $w_3 = 1$ (solid line) $\Rightarrow$ B-spline curve
- $w_3 = \frac{1}{2}$ (dashed line)



Periodic NURBS

How do we model a shape with NURBS?

idea: close the curve to a periodic one
$\Rightarrow C^{(k)}(a) = C^{(k)}(b),\ k = 0, \dots, d-p$

change definition of

- $N_{i,d}(u)$ repeat themselves periodically (shifted)
- change $P_i \Rightarrow$ periodic sequence

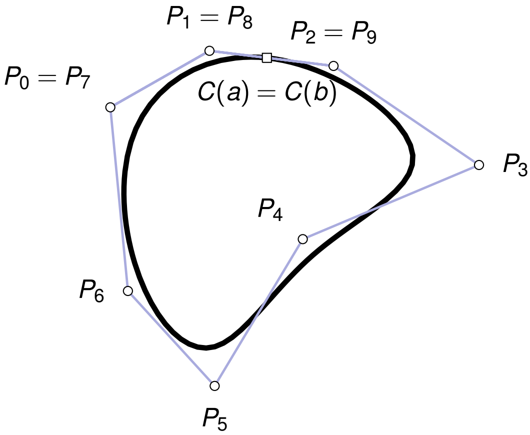
$\Rightarrow$ periodic curve

- no endpoint interpolation!
- **all** other properties remain.
- adapt algorithms, so they keep the curve periodic!

Periodic NURBS – example

What does s shape look like?

- smooth curve
- surrounds exactely one area iff it is injective.



- degree 3
- $C^{(k)}(a) = C^{(k)}(b)$, $k = 0, 1, 2$

Periodic NURBS – example

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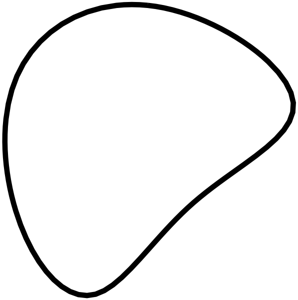

- degree 3
- $C^{(k)}(a) = C^{(k)}(b)$, $k = 0, 1, 2$
- start/end could be moved anywhere
- if any point on the curve may be moved

$\Rightarrow$ just the curve is needed in the GUI

Algorithms for NURBS

All your NURBS need are... these.

with these (periodic) NURBS the following algorithms were implemented

- calculate $C^{(k)}(u), k = 0, \ldots, m$ ($k = 0$ for drawing)
- extract arbitraty subcurve (ignoring start/end)
- affin linear transformations (affecting subcurve or whole curve)
- projection onto the curve (essential for interactive editing)
- moving arbitrary point on curve anywhere

except for projection, all these algorithms are well known.
$\Rightarrow$ small adjustments to fit periodic NURBS
$\Rightarrow$ for projection: circular clipping algorithm (Chen et al., 2008)

Drawing hypergraphs using NURBS curves

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Excursion: Distance betwenn a point and a NURBS Curve

Distance and projection on the NURBS Curve

Projection means, for a NURBS curve $C(u)$ (with control Points and weight P_i, w_i) and a point p
Compute the point $C(\hat{u})$ with shortest distance to p

The algorithm is important for interactive editing!
(point inversion)

It's idea is based on clipping using a circles around p, that get smaller and smaller

Main idea of the algorithm

divide and conquer with Newton iteration

main idea: circles around p of small radii, cut everything outside

1. Split the Curve into small parts (knot insertion $\Rightarrow$ beziér curves)
2. init circle around p running through $C(a)$ or $C(b)$
3. For each part decide whether
   - the part is outside the circle $\Rightarrow$ discard
   - the part is inside the circle, then
     - a) it has exactly one minimum $\Rightarrow$ Newton iteration $\Rightarrow$ new circle
     - b) it has more than one $\Rightarrow$ split in the middle, use endpoints as new circle radii
4. among all local minima from the newton iteration is the global minimum

Looking at the distance

calculating a function for the distance

The product

$$f(u) = (C(u) - p)(C(u) - p)^T$$

is the **objective squared distance function**.
$f(u)$ is a beziér curve iff $C(u)$ is a beziér curve (K. Mørken,1991)
degree of $f(u)$: $2d$

$f(u)$ has real valued control points!

$\Rightarrow$ use $f(u)$ to determine whether a curve is outise a circle around p (convex hull property)

$\Rightarrow$ if the control points have one "turning point" (variation diminishing property)
$\Rightarrow$ exactly one minimum

Projection – example

finding the shortest distance in a few slides

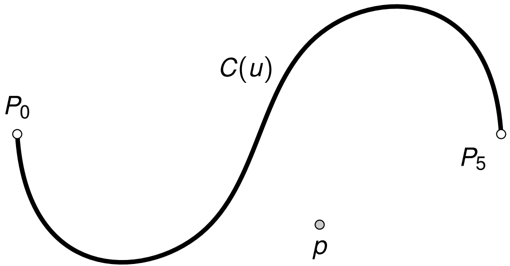


Figure: projection

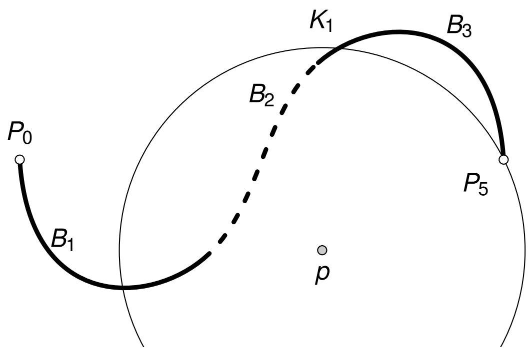
At first: split
Initial circle K_1

1. $B_1 \Rightarrow$ Circle K_2
   $\Rightarrow$ discard B_1
2. B_2 is inside K_2
   $\Rightarrow$ newton $\Rightarrow p_C$
3. new radius with p_C
   $\Rightarrow K_3$
4. discard B_3, it is outside K_3

only one newton iteration
result is p_C

Projection – example

finding the shortest distance in a few slides



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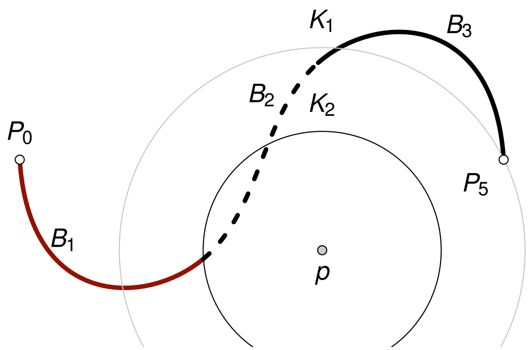

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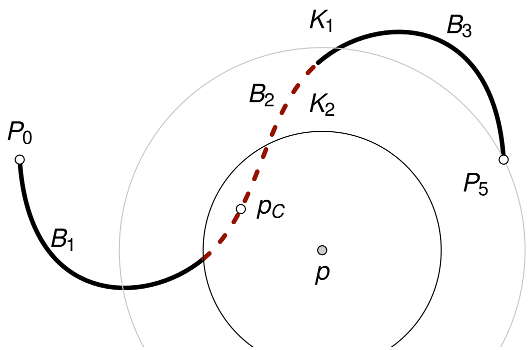

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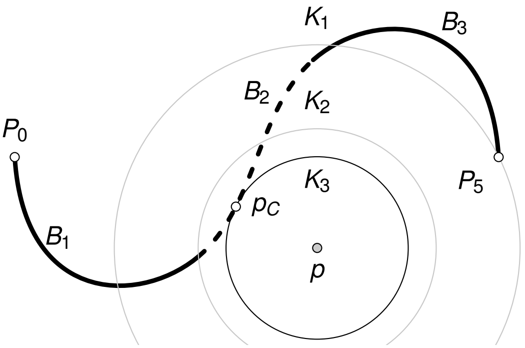

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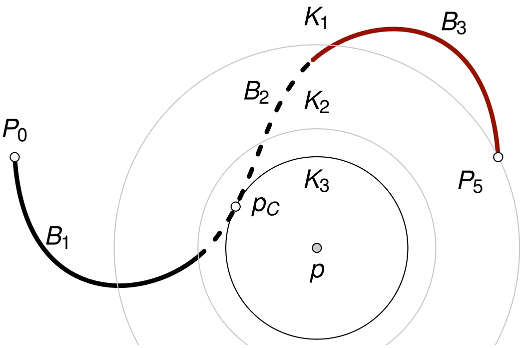

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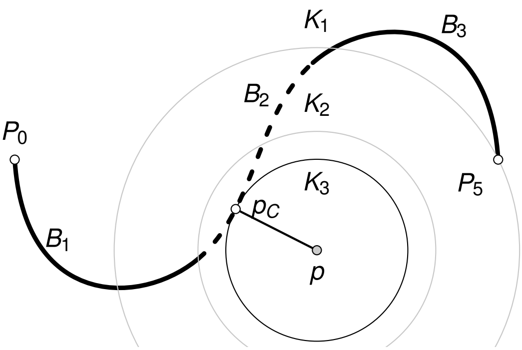
Projection – example

finding the shortest distance in a few slides



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3. new radius with p_C $\Rightarrow K_3$
4. discard B_3, it is outside K_3

only one newton iteration
result is p_C

The hyperedge shape

What should a drawing of a hyperedge look like? Part I

hypergraph $\mathscr{H} = (V, \mathscr{E})$, with nodes $v_i \in V$ and hyperedges $E_i \in \mathscr{E} \subset \mathscr{P}(V)\backslash\emptyset$

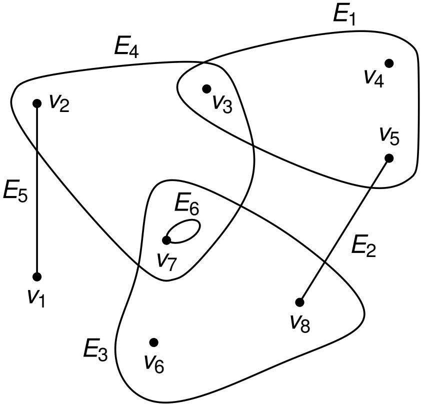


Figure: a Gravel export of a hypergraph by C.Berge

types of shapes

- loops – e.g. E_6
- iff $|E| = 2|$, simple curve joining the nodes, e.g. E_5
- periodic curve enclosing only its nodes, e.g. E_3

The hyperedge shape – decorations

What should a drawing of a hyperedge look like? Part II

additional attributes for the curve of a shape

- solid, dashed, dotted,...
- line width
- color
- label (cf. last slide)

and a margin δ:

shortest distance from node borders of each $v \in E$ to the curve

with $\delta > 0$ no node “touches” the curve
with $\delta > \alpha \in \mathbb{R}^+$ we have a margin inside the shape

blackboard

The hyperedge shape – construction

Creating a shape for an hyperedge.

Using periodic NURBS curves, we can create shapes:

- circles
- interpolation through user defined points
- convex hull based on interpolation and margin

and modify them globally or locally by

- scaling, rotation, translation
- move control points or positions on the curve
- replace parts

The hyperedge shape – validation

Did you forget including a node?

a hyperedge shape can be validated:

- Are all nodes $v \in E$ inside the shape?
- Are all others outside?
- Is the margin big enough?

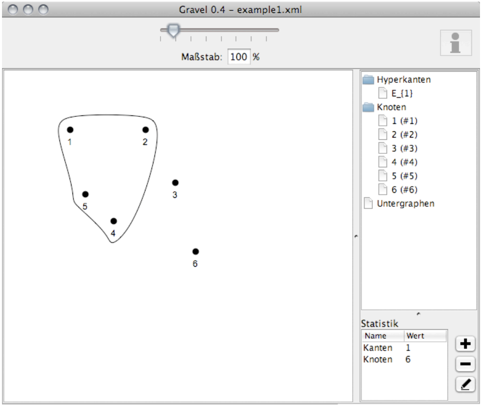
some criteria can't be checked (up to now?)

- minimization of crossings
- simplicity and other aesthetic criteria

Gravel – editing graphs and hypergraphs

So how can you use that now?

all the presented elements are implemented in **Gravel**, an editor for graphs and hypergraphs



Summary

Everything in short again

- hyperedges in the subset standard require periodic curves
- using NURBS and their algorithms
- extended to periodic NURBS

$\Rightarrow$ interactive editing

- hyperedge shape as formal definition of the hyperedge drawing
- easy creation and modification of a shape
- validation of the hyperedge shape (mostly) possible

$\Rightarrow$ a first editor for hypergraphs

future plans

What's next?

Gravel is available at gravel.darkmoonwolf.de (though in german only), the complete application is available as

- jar-file
- Mac OS X Application package
- source files

future plans are

- internationalization (using Java i18n)
- an algorithm API for
  - graph and hypergraph drawing algorithms
  - educative presentations of well known algorithms
  - stepwise execution of algorithms
- more basic shapes for hyperedges
- ...

One final example

TEX-Export using a TikZ picture in LATEX

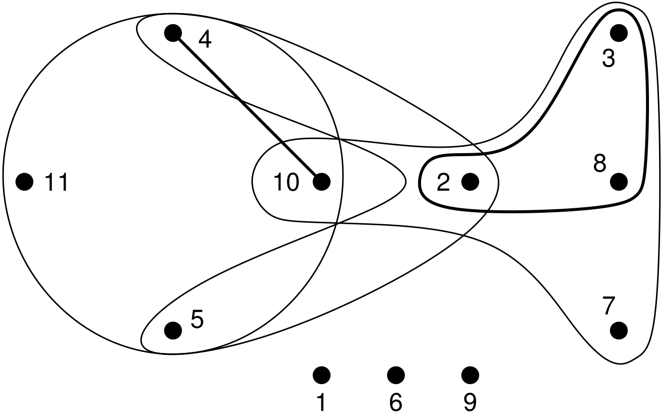


Figure: A competition hypergraph from Sonntag and Teichert

The End

Thanks for your attention.

Are there any questions?